1. $V'(t) = 2\pi rh \cdot r'(t) + \pi r^2 \cdot h'(t) = -320\pi\,\mathrm{cm^2/min}$ using the given information.

2. Information gives $r = \frac{2}{5}h$, and so therefore $V = \frac{4}{75}\pi h^3$. Then $V'(t) = \frac{4}{25}\pi h^2 \cdot h'(t)$. Given $h = 5$ and $V' = \pi\,\mathrm{m^3/min}$, solving gives $h' = \frac{1}{4}\mathrm{m/min}$, and then $r' = \frac{2}{5}h' = \frac{1}{10}\mathrm{m/min}$.

3. If John's distance to the island is j and Aeryn's is a, and distance between them is d, then $j^2 + a^2 = d^2$, so $2jj' + 2aa' = 2dd'$. Given $j = 120\text{km}$, $a = 50\text{km}$, $j' = -10\,\mathrm{km/h}$, $a' = 50\,\mathrm{km/h}$, then $d = 130\text{km}$, so $d' = +20\,\mathrm{km/h}$.

4. (a) $f' = 3(x-1)(x-3)$ so crit #s at $x = 1, 3$. Min of 0 at $x = 0, 3$ and max of 4 at $x = 1, 4$.
(b) $f' = 1 - 16/x^2$ so crit #s at $x = -4, 4$ (ignore -4). Min of 8 at $x = 4$, max of 17 at $x = 1$.
(c) $f' = x(x+2)e^x$ so crit #s at $x = -2, 0$. Min of 0 at $x = 0$, max of e at $x = 1$.
(d) $f' = 2 + 4\cos(x)$ so crit #s at $x = 2\pi/3$. Min of -2 at $x = \pi$, max of 6 at $x = 0$.
(e) $f' = \frac{5}{2}x^{3/2} - \frac{5}{2}x^{-1/2}$ so crit #s at $x = 0, 1$. Min of -4 at $x = 1$, max of $4\sqrt{3}$ at $x = 3$.
(f) $f' = (1-x^2)/(1+x^2)^2$ so crit #s at $x = -1, 1$. Min of $-1/2$ at $x = -1$, max of $1/2$ at $x = 1$.

5. Since f is continuous, $f(-1) = -1$, $f(0) = 1$, by IVT there is a real root in $(-1, 0)$. Since $f' = 3x^2 + 3$ is never zero, f cannot have 2 roots since by Rolle f' would be zero somewhere between them. So f has exactly 1 root.

6.

Critical #s	Infl #s	Increasing	Decreasing	Concave ↑	Concave ↓
2 (min)	None	$(2, \infty)$	$(-\infty, 2)$	$(-\infty, \infty)$	Never
-7 (max), 1 (min)	-3	$(-\infty, -7)$, $(1, \infty)$	$(-7, 1)$	$(-3, \infty)$	$(-\infty, -3)$
-4 (max), 0 (min)	(0), -3	$(-\infty, -4)$, $(0, \infty)$	$(-4, 0)$	$(-3, 0)$, $(0, \infty)$	$(-\infty, -3)$
$-1, 1$ (mins), 0 (max)	$\pm\frac{1}{\sqrt{3}}$	$(-1, 0)$, $(1, \infty)$	$(-\infty, -1)$, $(0, 1)$	$(-\infty, -\frac{1}{\sqrt{3}})$, $(\frac{1}{\sqrt{3}}, \infty)$	$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
$\frac{2\pi}{3}$ (max), $\frac{4\pi}{3}$ (min)	π	$(0, \frac{2\pi}{3})$, $(\frac{4\pi}{3}, 2\pi)$	$(\frac{2\pi}{3}, \frac{4\pi}{3})$	$(\pi, 2\pi)$	$(0, \pi)$
-2 (min), 2 (max)	0, $\pm 2\sqrt{3}$	$(-2, 2)$	$(-\infty, -2)$, $(2, \infty)$	$(-\infty, 0)$	$(0, \infty)$

7. (a) L'Hopital once, $L = 1/3$.
(b) Not a L'Hopital limit (denom not 0), plug in for $L = 0$.
(c) L'Hopital twice, $L = 2$.
(d) Common denominator then L'Hopital twice, $L = 0$.
(e) L'Hopital twice, $L = \infty$.
(f) Write as $\ln(x-2)/[1/(x^2-4)]$ then L'Hopital once, $L = 0$.
(g) Take natural log then write as $3\ln(1 + 2/x)/(1/x)$ then L'Hopital once and exponentiate, $L = e^6$.
(h) Take natural log then write as $4\ln(x^2 + 3x)/\ln(x)$ then L'Hopital twice and exponentiate, $L = e^8$.

8. Numbers are x, x, y with $2x + y = 12$ meaning that $y = 12 - 2x$. Then the product is $p(x) = x^2(12 - 2x)$ where we must have $0 < x < 6$. Only critical pt of p in that range is local max $x = 4$, so max product is $p(4) = 64$.

9. Let triangle sides be $3s$, $4s$, $5s$ with $0 \le s \le 1$, then square side is $3 - 3s$. Total area is $A(s) = 6s^2 + (3-3s)^2 = 15s^2 - 18s + 9$. Only critical point is $s = 3/5$. Min area is $18/5$ at $s = 3/5$ and max area is 9 at $s = 0$.

10. Box dimensions are s, $14-2s$, $30-2s$ so $V(s) = s(14-2s)(30-2s)$, with $0 \le s \le 7$. Then $V'(s) = 4(s-3)(3s-35)$ so the only critical point of V in that range is local max $s = 3$, which must be the absolute max.

11. Take antiderivatives and plug in to get $f'(x) = 4x^3 - e^x - \cos(x) + 3$, $f(x) = x^4 - e^x - \sin(x) + 3x + 3$.

12. Pop. is exponential: $P(t) = Ce^{kt}$, t yrs after 2010. Plug in to get $C = 500$ and $k = \ln(3)/10$, so $P(30) = 4500$.

13. (a) $RS_{\text{left}} = \sqrt{0}\cdot 2 + \sqrt{2}\cdot 2 \approx 2.828$, $RS_{\text{mid}} = \sqrt{1}\cdot 2 + \sqrt{3}\cdot 2 \approx 5.464$, $RS_{\text{right}} = \sqrt{2}\cdot 2 + \sqrt{4}\cdot 2 \approx 6.828$.
(b) $RS_{\text{left}} = 0.16$, $RS_{\text{mid}} = 0.245$, $RS_{\text{right}} = 0.36$.

14. (a) $232/9$ (b) $\sin^{-1}(x) + \tan^{-1}(x) + C$ (c) 3 (d) 0 (e) $\sqrt{2} - 1$ (f) $\frac{1}{2}(e^2 - 2e + 3)$
(g) $\sqrt{3}$ (h) 2 (i) $\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$ (j) $\ln(3)$ (k) $\frac{24}{\ln(3)} - \frac{6}{\ln(2)}$ (l) $x + C$

15. (a) 9 (b) -3 (c) 12 (d) 12