1. Find the following limits or explain why they do not exist (note ∞ and $-\infty$ are possible values):
   (a) $\displaystyle\lim_{x\to 1} \frac{\sin(\pi x)+e^x}{4\tan^{-1}(x)}$.
   (b) $\displaystyle\lim_{x\to 3+} \frac{x^2-9}{x^2+2x-15}$.
   (c) $\displaystyle\lim_{x\to 2-} \frac{\sqrt{x+7}-3}{x-2}$.
   (d) $\displaystyle\lim_{x\to 1} \frac{1}{(x-1)^6}$.
   (e) $\displaystyle\lim_{x\to 2-} \frac{x}{(x+1)(x-2)}$.
   (f) $\displaystyle\lim_{x\to\infty} \frac{x^3+2x^2+5}{2x^2-x+1}$.
   (g) $\displaystyle\lim_{x\to\infty} \frac{3x^5+x^3}{2x^6-11}$.
   (h) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2-x+2}}{x^2+1}$.

2. Use the Intermediate Value Theorem to show that if $f(x) = x^{20} + 3x^{19} + 1$, then there must exist a real number c with $f(c) = 2$.

3. If $f(x) = 2x^2 + 2$, find the average rate of change of f on the intervals (a) $[1, 3]$, (b) $[1, 2]$, and (c) $[1, 1.1]$.

4. Use the limit definition of the derivative to find (a) $f'(x)$ for $f(x) = x^2 + 1$, and (b) $s'(1)$ for $s(t) = \sqrt{3t+1}$.

5. Find the following derivatives:
   (a) $\displaystyle\frac{d}{dx}\left[2x^3 + \sqrt{x} + \frac{4}{x\sqrt[3]{x}} + 5\right]$.
   (b) $\displaystyle\frac{dy}{dx}$ if $y = x^{56}\cos(x)\sqrt{1-x^2}$.
   (c) $\displaystyle\frac{d^2 f}{dx^2}$ if $f(x) = x^3 2^x$.
   (d) $\displaystyle\frac{dg}{dy}$ if $g(y) = \frac{y\sin^{-1}(y)}{\cos(4y)-\ln(y)}$.
   (e) $\displaystyle\frac{d}{dt}\left[\sqrt{\ln(\sin(t))}\right]$.
   (f) $f''(x)$ if $f(x) = \tan^{-1}(x)$.
   (g) $f^{(2019)}(x)$ for $f(x) = \cos(4x)$.
   (h) $x'(y)$ if $x = \displaystyle\frac{4+\sqrt{3-\tan^{-1}(y)}}{e^{2y}\sin(5y)}$.
   (i) $\triangle'(\star)$ where $\triangle(\star) = \square^{\star}$.

6. Use logarithmic differentiation to find the derivative of each function:
   (a) $f(x) = \sqrt{(\sin x)^{\cos x}}$.
   (b) $g(x) = \displaystyle\frac{e^{-\cos(x)}\cdot[\ln(x)]^{3x}}{(\tan^{-1}(x))^2}$.
   (c) $h(x) = \displaystyle\frac{x^4}{(x^2-2x+5)^8\cdot\sqrt[3]{5x+1}}$.

7. Find an equation for the tangent line to the graph of $y = 2\sin(3x)$ at $x = \pi/4$.

8. Consider the implicit curve $x^2y + x^5y^6 = 1$, which defines y implicitly as a function of x.
   (a) Find dy/dx.
   (b) Find an equation for the line tangent to the curve at the point $(x, y) = (1, 1)$.

9. If x and y are related by $20x^5 - xy^4 + e^{2x-y} = 5$, find dy/dx when $x = 1$ and $y = 2$.

10. A particle moves through the plane so that at time t seconds, its position is $(x, y) = (3e^{2t}, e^{6t})$ meters.
    (a) Find the particle's velocity, speed, and acceleration at time t.
    (b) Find dy/dx at time t.
    (c) Find an equation for the tangent line to the particle's path at time $t = 0$.

11. Let $f(x) = 7x^4 + 2x + 1$ and $g(x) = \arctan(x^2)$. Find the linearizations of $f(x)$ and $g(x)$ at $x = 1$.

12. Using appropriate linear approximations, estimate the values of $e^{0.04}$ and $\sqrt[5]{32.08}$.

13. The function $f(x) = 4x + \sin(3x)$ is one-to-one, so it has an inverse function $g(x)$. Find $g(4\pi)$ and $g'(4\pi)$.

14. Suppose that $f(1) = 5$, $f'(1) = 6$, $f(5) = 5$, $f'(5) = 2$, $g(1) = 5$, and $g'(1) = 8$. Find the following:
    (a) The derivative of $f(x)\cdot g(x)$ at $x = 1$.
    (b) The derivative of $f(x)/g(x)$ at $x = 1$.
    (c) The derivative of $f(g(x))$ at $x = 1$.
    (d) The derivative of $f(f(f(x)))$ at $x = 1$.

15. Suppose that $f(x) = \begin{cases} 3x + c & \text{for } x \le 0 \\ dx^2 - 1 & \text{for } x > 0 \end{cases}$. Determine all possible values of c and d so that (a) f is continuous for all real numbers x, and (b) f is differentiable for all real numbers x.