

1. (a) e^π/π . (b) -3 . (c) $1/6$. (d) ∞ . (e) $-\infty$. (f) ∞ . (g) 0 . (h) 0 .

2. Since f is a polynomial, it is continuous. Also since $f(0) = 1$ and $f(1) = 4$, and $1 < 2 < 4$, the Intermediate Value Theorem says that there must exist a real number c in the interval $(0, 1)$ with $f(c) = 2$.

3. (a) $\frac{f(3) - f(1)}{3 - 1} = 8$. (b) $\frac{f(2) - f(1)}{3 - 1} = 6$. (c) $\frac{f(1.1) - f(1)}{1.1 - 1} = 4.2$.

4. (a) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 1] - (x^2 + 1)}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x$.
 (b) $s'(1) = \lim_{h \rightarrow 0} \frac{\sqrt{3(1+h)+1} - 2}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{3h+4} - 2}{h} \cdot \frac{\sqrt{3h+4} + 2}{\sqrt{3h+4} + 2} = \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{3h+4} + 2)} = \lim_{h \rightarrow 0} \frac{3}{\sqrt{3h+4} + 2} = \frac{3}{4}$.

5. (a) $6x^2 + \frac{1}{2}x^{-1/2} + 4(-\frac{4}{3}x^{-7/3})$. (b) $56x^{55} \cos(x)\sqrt{1-x^2} - x^{56} \sin(x)\sqrt{1-x^2} + x^{56} \cos(x) \cdot \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)$.
 (c) $3x^2 \cdot 2^x + x^3 \cdot 2^x \ln(2)$. (d) $\frac{[1 \cdot \sin^{-1}(y) + y \cdot \frac{1}{\sqrt{1-y^2}}] \cdot [\cos(4y) - \ln(y)] - y \sin^{-1}(y) \cdot [-4 \sin(4y) - 1/y]}{[\cos(4y) - \ln(y)]^2}$.
 (e) $\frac{1}{2}[\ln(\sin(t))]^{-1/2} \cdot \frac{\cos(t)}{\sin(t)}$. (f) $f'(x) = \frac{1}{x^2 + 1}$, $f''(x) = \frac{-2x}{(x^2 + 1)^2}$. (g) $4^{2019} \sin(4x)$.
 (h) $\frac{\frac{1}{2}[3 - \tan^{-1}(y)]^{-1/2} \cdot \frac{1}{y^2+1} - [4 + \sqrt{3 - \tan^{-1}(y)}] \cdot [2e^{2y} \sin(5y) + e^{2y} \cdot 5 \cos(5y)]}{[e^{2y} \sin(5y)]^2}$. (i) $\square^{2^x} \ln(\square)$.

6. (a) $\ln(f) = \frac{1}{2} \cos(x) \cdot \ln(\sin(x))$ so $f' = \sqrt{(\sin x)^{\cos x}} \left[-\frac{1}{2} \sin(x) \ln(\sin(x)) + \frac{1}{2} \cos(x) \cdot \frac{\sin(x)}{\cos(x)} \right]$.
 (b) $\ln(g) = 4 \ln(x) - 8 \ln(x^2 - 2x + 5) - \frac{1}{3} \ln(5x + 1)$ so $h' = \frac{x^4}{(x^2 - 2x + 5)^8 \cdot \sqrt[3]{5x + 1}} \cdot \left[\frac{4}{x} - 8 \frac{2x - 2}{x^2 - 2x + 5} \right] - \frac{1}{3} \cdot \frac{5}{5x + 1}$.
 (c) $\ln(h) = -\cos(x) + 19 \ln(x^3 + 3) - 2 \ln(\tan^{-1}(x))$, so $g' = \frac{e^{-\cos(x)} \cdot (x^3 + 3)^{19}}{(\tan^{-1}(x))^2} \cdot \left[\sin(x) + 19 \cdot \frac{3x^2}{x^3 + 3} - 2 \cdot \frac{1/(1+x^2)}{\tan^{-1}(x)} \right]$.

7. Since $y' = 6 \cos(3x)$, point is $(\pi/4, \sqrt{2})$ and slope is $y'(\pi/4) = -3\sqrt{2}$, so equation is $y - \sqrt{2} = -3\sqrt{2}(x - \pi/4)$.

8. (a) Differentiating yields $2xy + x^2y' + 2 \cdot 5x^4y^6 + 2x^5 \cdot 6y^5y' = 0$ so $y' = \frac{dy}{dx} = -\frac{2xy + 10x^4y^6}{x^2 + 12x^5y^5}$.
 (b) Slope is $\frac{dy}{dx}$ at $(x, y) = (1, 1)$, which is $-\frac{12}{13}$. Equation is $y - 1 = -\frac{12}{13}(x - 1)$.

9. Differentiating, $100x^4 - (y^4 + x \cdot 4y^3y') + e^{2x-y}(2 - y') = 0$. Hence $y' = \frac{100x^4 - y^4 + 2e^{2x-y}}{4xy^3 + e^{2x-y}}$. At $(1, 2)$, $y' = -\frac{86}{33}$.

10. (a) Velocity is $(x', y') = (6e^{2t}, 6e^{6t})$, speed is $\sqrt{(6e^{2t})^2 + (6e^{6t})^2}$, acceleration is $(x'', y'') = (12e^{2t}, 36e^{6t})$.
 (b) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{6e^{6t}}{6e^{2t}} = e^{4t}$. (c) Point $(x(0), y(0)) = (3, 1)$, slope $dy/dx = 1$, equation $y - 1 = 1(x - 3)$.

11. (f) Since $f(1) = 10$, $f'(1) = 30$, linearization is $L(x) = 10 + 30(x - 1)$.
 (g) Since $g(1) = \pi/4$, $g'(1) = 2$, linearization is $L(x) = \pi/4 + 2(x - 1)$.

12. (a) Linearize $f(x) = e^x$ at $x = 0$ yields $L(x) = 1 + x$. Estimate is $L(0.04) = 1.04$.
 (b) Linearize $g(x) = \sqrt[5]{x} = x^{1/5}$ at $x = 32$ yields $L(x) = 2 + \frac{1}{80}(x - 32)$. Estimate is $L(32.08) = 2.001$.

13. Since $f(\pi) = 4\pi$ we see $g(4\pi) = f^{-1}(4\pi) = \pi$. Note $f'(x) = 4 + 3 \cos(3x)$.
 Then by the formula $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ this gives $g'(4\pi) = \frac{1}{f'(f^{-1}(4\pi))} = \frac{1}{f'(\pi)} = \frac{1}{4 + 3 \cos(3\pi)} = 1$.

14. (a) This is $f'(1)g(1) + f(1)g'(1) = 70$. (b) This is $[f'(1)g(1) - f(1)g'(1)]/[g(1)]^2 = -2/5$.
 (c) This is $f'(g(1)) \cdot g'(1) = f'(5) \cdot 8 = 16$. (d) This is $f'(f(f(1))) \cdot f'(f(1)) \cdot f'(1) = 3 \cdot 2 \cdot 6 = 36$.

15. (a) $c = -1$ and d is arbitrary. (b) $c = -1$ and $d = 3/2$.