1. (a) Sub $u = 4x + 1$, result $(e^5 - e)/4$.
   (b) Sub $u = 2x$, result $(5/2)(\tan^{-1} 2 - \tan^{-1} 0)$.
   (c) Sub $u = 1 + 4x^2$, result $(5/8)[\ln 5 - \ln 1]$.
   (d) Sub $u = 6z$, result $[\sec \frac{\pi}{3} - \sec \frac{\pi}{6}]/6$.
   (e) Expand out, result $\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C$.
   (f) Sub $u = t^2 + 1$, result $\frac{1}{6}(t^2 + 1)^3 + C$.
   (g) Sub $u = -2x$, result $-\frac{1}{2 \ln 8} \cdot [8^{-2} - 8^{-0}]$.
   (h) Sub $u = \tan x$, result $\frac{1}{4}\tan^4 x + C$.
   (i) Sub $u = \sec x$, result $\frac{1}{6}\sec^6 x + C$.
   (j) Sub $u = 2x + 5$, result $-\frac{1}{6}(2x + 5)^{-3} + C$.
   (k) Sub $u = 5 + e^y$, result $\frac{2}{3}(5 + e^y)^{3/2} + C$.
   (l) Expand out, result $\frac{1}{3}x^3 + \ln x + C$.
   (m) Sub $u = \sin(2x)$, result $\frac{1}{2}(e - 1)$.
   (n) Sub $u = 1/x$, result $-4(e^{1/2} - e^1)$.
   (o) Sub $u = x + 4$, result $\frac{2}{5}(x + 4)^{5/2} - \frac{8}{3}(x + 4)^{3/2} + C$.
   (p) Sub $u = x^{3/2}$, result $-\frac{2}{3}\cos(x^{3/2}) + C$.
   (q) Sub $u = x^2 + a^2$, result $\frac{2}{3}[(2a^2)^{3/2} - (a^2)^{3/2}]$.
   (r) Sub $u = \cos x$, result $-\frac{2}{5}[\cos^{5/2} \frac{\pi}{2} - \cos^{5/2} 0] = \frac{2}{5}$.
   (s) Sub $u = 1 - t$, result $4(1 - t)^{1/2} - \frac{4}{3}(1 - t)^{3/2} + C$.
   (t) Sub $u = \sin x$, result $5\tan^{-1}(\sin x) + C$.
   (u) Sub $u = e^x - e^{-x}$, result $\ln(e^x - e^{-x}) + C$.
   (v) Function is $e^{15x} - e^{14x}$, res. $\frac{1}{15}(e^{15} - 1) - \frac{1}{14}(e^{14} - 1)$.
   (w) Sub $u = 1 + \ln x$, result $\ln 2 - \ln 1$.
   (x) Sub $u = x^4 + x^2 + 1$, result $-\frac{1}{14}[3^{-7} - 1^{-7}]$.
   (y) Sub $u = x^2 + 4x + 6$, result $\frac{1}{4}e^{x^2+4x+6} + C$.
   (z) Sub $x = u^2$, result $4(\ln 4 - \ln 3)$.

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2. (a) By FTOC, this is e^{x^3}.
   (b) Write $f(x) = \int_0^x e^{t^3}\,dt$ where $f'(x) = e^{x^3}$ by FTOC. Then the given integral is $f(\tan x) - f(\sin x)$ with derivative $f'(\tan x) \cdot \sec^2 x - f'(\sin x) \cdot \cos x = e^{\tan^3 x} \sec^2 x - e^{\sin^3 x} \cos x$.
   (c) Write $f(x) = \int_0^x \frac{\sin t}{t+2}\,dt$ where $f'(x) = \frac{\sin x}{x+2}$ by FTOC. Then the given integral is $f(3x) - f(2x)$ with derivative $f'(3x) \cdot 3 - f'(2x) \cdot 2 = \frac{\sin 3x}{3x+2} \cdot 3 - \frac{\sin 2x}{2x+2} \cdot 2$.

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3. (a) Average value is $\frac{1}{9-4}\int_4^9 \sqrt{x}\,dx = \frac{2}{15}(27 - 8) = \frac{38}{15}$.
   (b) Average value is $\frac{1}{\pi/4 - 0}\int_0^{\pi/4} \sin(2x)\,dx = \frac{1}{\pi/4} \cdot \frac{1}{2} = \frac{2}{\pi}$ (evaluate the integral by substituting $u = 2x$).

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4. (a) Function is positive for $3 \le x \le 4$ so area is $\int_3^4 (x^2 - 5)\,dx = \frac{22}{3}$.
   (b) Function is above x-axis when $0 \le x \le 2$ so area is $\int_0^2 (2x - x^2)\,dx = \frac{4}{3}$.
   (c) Function is below x-axis when $-1 \le x \le 1$ so area is $\int_{-1}^1 -(1 - x^2)\,dx = \frac{4}{3}$.
   (d) Curves cross at $x = 1, 4$ and line is above, so area is $\int_1^4 [5x - (x^2 + 4)]\,dx = \frac{9}{2}$.
   (e) Curves cross at $x = \pm 2$ and $3 - x^2$ is above, so area is $\int_{-2}^2 [(3 - x^2) - (x^2 - 1)\,dx = \frac{16}{3}$.
   (f) Curve intersects axes at $y = 0, 2$, so area is $\int_0^2 (4 - y^2)\,dy = \frac{16}{3}$.
   (g) Curve intersects y-axis at $y = 0$, so area is $\int_0^1 y^6\,dy = \frac{1}{7}$.
   (h) Region is $0 \le x \le 2$ and curves cross at $x = 1$, so area is $\int_0^1 (x + 2)\,dx + \int_1^2 (4 - x^2)\,dx = \frac{25}{6}$.
   (i) Region is $-2 \le y \le 1$ and curves do not cross (ever), so area is $\int_{-2}^1 [(y^2 + 1) - (y^2 - 2)]\,dy = 9$.

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5. Information says that $\int_0^a f(x)\,dx = a^3 + e^a - 1$. Differentiating both sides with respect to a and applying FTOC gives $f(a) = \frac{d}{da}[a^3 + e^a - 1] = 3a^2 + e^a$ (which does work).
