Life is about using the whole box of crayons! – RuPaul

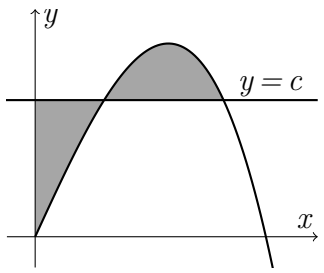
Problem 1 (Putnam 1962. B4)**.** *The Euclidean plane is divided into regions by drawing a finite number of circles. Show that it is possible to color each of these regions either red or blue in such a way that no two adjacent regions have the same color. (Two such regions are said to be adjacent if and only if their boundaries have an arc of a circle in common.)*

Problem 2 (Putnam 1964. B6)**.** *Show that the points of the unit disc cannot be colored red and green in such a way that the set of red points is congruent to the set of green points.*

Problem 3 (Putnam 1988. A4)**.** *(a) If every point of the plane is painted one of three colors, do there necessarily exist two points of the same color exactly one inch apart?*

(b) What if "three" is replaced by "nine"?

Problem 4 (Putnam 1993. A1)**.** *The horizontal line $y = c$ intersects the curve $y = 2x - 3x^3$ in the first quadrant as in the figure. Find c so that the areas of the two shaded regions are equal.*



Problem 5 (Putnam 1994. A3)**.** *Show that if the points of an isosceles right triangle of side length 1 are each colored with one of four colors, then there must be two points of the same color which are at least a distance $2 - \sqrt{2}$ apart.*

Problem 6 (Putnam 2022. B3)**.** *Assign to each positive real number a color, either red or blue. Let D be the set of all distances $d > 0$ such that there are two points of the same color at distance d apart. Recolor the positive reals so that the numbers in D are red and the numbers not in D are blue. If we iterate this recoloring process, will we always end up with all the numbers red after a finite number of steps?*

Hints:

1. Try choosing the coloring based on how many circles a region lies inside.
2. Such a coloring exists if you exclude the center. So let the center be O with O' the corresponding opposite-color point: consider the diameter perpendicular to OO'.
3. (a) (b) Try tiling the plane with squares labeled 1-9.
4. Express the given condition in terms of an integral, then evaluate the integral.
5. Starting with the vertices, look about points that are a distance $2 - \sqrt{2}$ from each other.
6.