Play it cool, that's the old school rule man. – Nas

Problem 1 (Putnam 1939. B7). *Prove that* $u^3 + v^3 + w^3 - 3uvw = 1$, *where*

$$u = 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \cdots, \qquad v = \frac{x}{1!} + \frac{x^4}{4!} + \frac{x^7}{7!} + \cdots, \qquad w = \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^8}{8!} + \cdots.$$

Problem 2 (Putnam 1942. A1). *A square of side* $2a$, *lying always in the first quadrant of the* xy*-plane, moves so that two consecutive vertices are always on the* x*- and* y*-axes respectively. Find the locus of the center of the square.*

Problem 3 (Putnam 1947. B4). *Let* $P(z) = z^2 + az + b$ *be a quadratic polynomial of the complex variable* z *with complex coefficients* a *and* b. *Suppose that* $|P(z)| = 1$ *for every* z *such that* $|z| = 1$. *Prove that* $a = b = 0$.

Problem 4 (Putnam 1948. A3). *Let* $\{a_n\}$ *be a decreasing sequence of positive numbers with limit 0 such that* $b_n = a_n - 2a_{n+1} + a_{n+2} \geq 0$ *for all* n. *Prove that* $\sum_{n=1}^{\infty} nb_n = a_1$.

Problem 5 (Putnam 1948. B4). *For which real numbers* λ *does the equation* $\int_0^1 \min(x, y)f(y)\,dx = \lambda f(x)$ *have a continuous solution that does not vanish identically on* $[0, 1]$*? What are these solutions?*

Problem 6 (Putnam 1949. B1). *Each rational number* p/q *in lowest terms in the interval* $(0, 1)$ *is covered by a closed interval of length* $1/(2q^2)$ *whose center is at* p/q. *Prove that* $\sqrt{2}/2$ *is not covered by any of these closed intervals.*

Hints:

1. Try differentiating the relation that you're supposed to prove.
2. Parametrize the motion in terms of the angle formed by the side and the x-axis.
3. Compute $\int_0^{2\pi} |p(e^{it}|^2\, dt$.
4. Let $c_n = a_n - a_{n+1}$ and show $b_n = c_n - c_{n+1}$, then telescope.
5. Break the integration range into $[0, x]$ and $[x, 1]$, then differentiate both sides.
6. Suppose that $|\sqrt{2}/2 - p/q| < 1/(4q^2)$. Try to prove that $|2p^2 - q^2| < 1$.