Life is more fun if you play games. – Roald Dahl

Problem 1 (Putnam 1989. B1)**.** *A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge.*

Problem 2 (Putnam 1993. B2)**.** *Consider the following game played with a deck of* $2n$ *cards numbered from 1 to* $2n$*. The deck is randomly shuffled and* n *cards are dealt to each of two players. Beginning with* A*, the players take turns discarding one of their remaining cards and announcing its number. The game ends as soon as the sum of the numbers on the discarded cards is divisible by* $2n+1$*. The last person to discard wins the game. Assuming optimal strategy by both* A *and* B*, what is the probability that* A *wins?*

Problem 3 (Putnam 2001. B1)**.** *Let* n *be an even positive integer. Write the numbers* $1, 2, \ldots, n^2$ *in the squares of an* $n \times n$ *grid so that the* k*-th row, from left to right, is* $(k-1)n+1, (k-1)n+2, \ldots, (k-1)n+n$*. Color the squares of the grid so that half of the squares in each row and in each column are red and the other half are black (a checkerboard coloring is one possibility). Prove that for each coloring, the sum of the numbers on the red squares is equal to the sum of the numbers on the black squares.*

Problem 4 (Putnam 2008. A2)**.** *Alan and Barbara play a game in which they take turns filling entries of an initially empty* 2008×2008 *array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?*

Problem 5 (Putnam 2009. B2)**.** *A game involves jumping to the right on the real line. If* a *and* b *are real numbers and* $b > a$*, the cost of jumping from* a *to* b *is* $b^3 - ab^2$*. For what real numbers* c *can one travel from* 0 *to* 1 *in a finite number of jumps with total cost exactly* c*?*

Problem 6 (Putnam 2010. B3)**.** *There are 2010 boxes labeled* $B_1, B_2, \ldots, B_{2010}$*, and* $2010n$ *balls have been distributed among them, for some positive integer* n*. You may redistribute the balls by a sequence of moves, each of which consists of choosing an* i *and moving* exactly i *balls from box* B_i *into any one other box. For which values of* n *is it possible to reach the distribution with exactly* n *balls in each box, regardless of the initial distribution of balls?*

Hints:

1. Draw the region and use symmetry.
2. Someone must win (why?). On any given move, how many "winning" cards are there?
3. Label the columns with $1, 2, \cdots, n$ and the rows with $0, n, 2n, \cdots, (n-1)n$.
4. What's an easy way to make a determinant of a matrix equal to zero?
5. If the intermediate steps are $\Delta x_1, \cdots, \Delta x_n$, what's the total cost?
6. If there are $i-1$ balls in each box, then there are no moves possible. But if some box has at least i....