Kids these days need to do more calculus. – Prof. Dummit

Problem 1 (Putnam 1980. B1). *For which real c is $(e^x + e^{-x})/2 \leq e^{cx^2}$ for all real x?*

Problem 2 (Putnam 1981. A3). *Find* $\lim_{t\to\infty}\left[e^{-t}\int_0^t\int_0^t \frac{e^x - e^y}{x-y}\,dx\,dy\right]$.

Problem 3 (Putnam 1982. A2). *For positive real x let $B_n(x) = 1^x + 2^x + 3^x + \cdots + n^x$. Prove or disprove the convergence of* $\sum_{n=2}^{\infty} \frac{B_n(\log_n 2)}{(n\log_2 n)^2}$.

Problem 4 (Putnam 1983. A2). *The hands of an analog clock have lengths 3 and 4. Find the distance between the tips of the hands when that distance is increasing most rapidly.*

Problem 5 (Putnam 1987. B1). *Evaluate* $\int_2^4 \frac{\sqrt{\ln(9-x)}\,dx}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}}$.

Problem 6 (Putnam 1992. B4). *Let $p(x)$ be a nonzero polynomial of degree less than 1992 having no nonconstant factor in common with $x^3 - x$. Let* $\frac{d^{1992}}{dx^{1992}}\left(\frac{p(x)}{x^3-x}\right) = \frac{f(x)}{g(x)}$ *for polynomials $f(x)$ and $g(x)$. Find the smallest possible degree of $f(x)$.*

Hints:

1. Try some values of x. Which x are the most restrictive on the condition?
2. Use L'Hopital's rule on the limit.
3. Find upper and lower bounds on $B_n(\log_n 2)$.
4. Use the law of cosines.
5. Use symmetry.
6. Use partial fraction decomposition.