**FALL 2026 PUTNAM PRACTICE**
**PROBLEM SET 3**

**Problem 1** (Putnam 1996 A1)**.** Find the least number $A$ with the following property: Whenever two squares have a total area of 1, they can be packed, without overlap of their interiors, into a rectangle of area $A$, with the sides of the squares parallel to the sides of the rectangle.

**Problem 2** (Putnam 2000 A1)**.** Let $A > 0$. What are the possible values of $\sum_{j=0}^{\infty} x_j^2$ given that $x_0, x_1, \ldots$ are positive real numbers for which $\sum_{j=0}^{\infty} x_j = A$?

**Problem 3** (Putnam 2004 B2)**.** For positive integers $m$ and $n$, prove that

$$\frac{(m+n)!}{(m+n)^{m+n}} < \frac{m!}{m^m}\frac{n!}{n^n}.$$

**Problem 4** (Putnam 1996 B2)**.** For every positive integer $n$, prove that

$$\left(\frac{2n-1}{e}\right)^{(2n-1)/2} < 1 \cdot 3 \cdot 5 \cdots (2n-1) < \left(\frac{2n+1}{e}\right)^{(2n+1)/2}.$$

**Problem 5** (Putnam 2018 A3)**.** Determine the maximum possible value of $\sum_{i=1}^{10} \cos(3x_i)$ subject to $\sum_{i=1}^{10} \cos(x_i) = 0$, where $x_1, \ldots, x_{10} \in \mathbb{R}$.

**Problem 6** (Putnam 1996 B3)**.** Suppose that $\{x_1, \ldots, x_n\} = \{1, 2, \ldots, n\}$. Determine the largest possible value of

$$x_1x_2 + x_2x_3 + \cdots + x_{n-1}x_n + x_nx_1.$$

**Problem 7** (Putnam 1979 A6)**.** Let $p_1, \ldots, p_n$ be real numbers satisfying $0 \leq p_i \leq 1$ $(1 \leq i \leq n)$. Prove that there exists $x \in [0, 1]$ such that

$$\sum_{i=1}^{n} \frac{1}{|x - p_i|} \leq 8n\left(1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}\right).$$

(1) Let the sides of the two squares be $\cos(t)$ and $\sin(t)$ respectively ($t \in [0, \frac{\pi}{4}]$).
(2) Use $\sum_j x_j^2 \leq (\sum_j x_j)^2$.
(3) Use the binomial theorem for the expression $(m+n)^{m+n}$.
(4) Take the logarithm and compare a sum with integrals.
(5) Express everything in $y_1, y_2, \ldots, y_{10}$ where $y_i = \cos(x_i)$.
(6) How much can the sum increase when going from $n$ to $n+1$?
(7) You are on your own.