FALL 2026 PUTNAM PRACTICE
PROBLEM SET 1

Problem 1 (Putnam 1971, A1)**.** Given any 9 distinct points in $\mathbb{Z}^2 \subset \mathbb{R}^2$, show that we can find two which have a lattice point on the interior of the segment joining them.

Problem 2 (Putnam 2002, A2)**.** Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

Problem 3 (Putnam 1953, A2)**.** Six points are in general position in space. Join every pair by a segment and color each segment red or blue. Prove that there is a triangle whose three sides have the same color.

Problem 4 (Putnam 1994, A4)**.** Let A and B be 2×2 matrices with integer entries. Suppose that

$$A, \quad A+B, \quad A+2B, \quad A+3B, \quad A+4B$$

are all invertible and that their inverses have integer entries. Prove that $A+5B$ is invertible and that its inverse has integer entries.

Problem 5 (Putnam 1966, B4)**.** Given a set S of $mn+1$ distinct positive integers, prove that we can either

(1) find $m+1$ elements $b_1, \ldots, b_{m+1} \in S$ such that b_i does not divide b_j for $i \neq j$, **or**
(2) find distinct integers $a_1, a_2, \ldots, a_{n+1} \in S$ such that a_i divides a_{i+1} for all i.

(Or both.)

Problem 6 (Folklore?)**.** Consider the set

$$X(n) = \{(a,b) \in \mathbb{Z}^2 \mid a, b \geq 1 \text{ and } a+b \leq n\}.$$

Show that this set cannot be covered by $n-1$ lines.

Problem 7 (IMO 2025, Day 1, 1)**.** Let $X(n)$ be defined as in Problem 7. A line is called *sunny* if it is not parallel to one of the lines $x=0$, $y=0$, $x+y=0$. For which k is it possible to cover $X(n)$ (as defined in Problem 7) with n lines such that exactly k of the lines are sunny.

(1) Consider the coordinates modulo 2.
(2) Consider a great circle (that divides the sphere into two equal hemisphere) that goes through 2 points.
(3) Choose one of the points and suppose that at least 3 of the 5 adjacent edges are red. Now consider the corresponding neighbors.
(4) Consider the polynomial $f(t) = \det(A + tB)$.
(5) For an element $a \in S$, define $f(a)$ as the largest integer k for which there exists a sequence $a_1 = a, a_2, \ldots, a_k$ of distinct elements of S with $a_1|a_2|\cdots|a_k$.
Assuming that $f(a) \leq n$ for all $a \in S$, show that we are in case (1).
(6) Assume that n lines cover $X(n)$ and show that one of the lines must be $x = 0$.
(7) Consider the case $n = 2$.