Every moment is a new beginning. – Elie Wiesel

Problem 1 (Putnam 2005. A1)**.** *Show that every positive integer is a sum of one or more numbers of the form $2^r 3^s$, where r and s are nonnegative integers and no summand divides another. (For example, $23 = 9 + 8 + 6$.)*

Problem 2 (Putnam 2014. A1)**.** *Prove that every nonzero coefficient of the Taylor series of $(1 - x + x^2)e^x$ about $x = 0$ is a rational number whose numerator (in lowest terms) is either 1 or a prime number.*

Problem 3 (Putnam 1967. B1)**.** *Let $ABCDEF$ be a hexagon inscribed in a circle of radius r. Show that if $AB = CD = EF = r$ then the midpoints of BC, DE, FA are the vertices of an equilateral triangle.*

Problem 4 (Putnam 2012. A1)**.** *Let $d_1, d_2, \ldots, d_{12}$ be real numbers in the open interval $(1, 12)$. Show that there exist distinct indices i, j, k such that d_i, d_j, d_k are the side lengths of an acute triangle.*

Problem 5 (Putnam 1986. A1)**.** *Find, with explanation, the maximum value of $f(x) = x^3 - 3x$ on the set of all real numbers x satisfying $x^4 + 36 \leq 13x^2$.*

Problem 6 (Putnam 2006. B1)**.** *Show that the curve $x^3 + 3xy + y^3 = 1$ contains only one set of three distinct points, A, B, and C, which are vertices of an equilateral triangle, and find its area.*

Hints:

1. Try it out for some small values of n. Are there any patterns? Are there any simple ways to use an expression for n to construct expressions for other values?

2. Using the Taylor series $e^x = \sum_{n=0}^{\infty} x^n/n!$, you can work out the Taylor series for the given function.

3. If O is the center of the circle, then ABO, CDO, and EFO are equilateral triangles. Try placing everything in the complex plane with O at the origin.

4. If not, then $d_{i-2}^2 + d_{i-1}^2 \geq d_i^2$ for each i. Based on this, what can you say about d_{12} in comparison to d_1?

5. When does $x^4 + 16 = 13x^2$? (Solve the quadratic equation for x^2.)

6. Use the identity $a^3 + b^3 + c^3 - 3abc = (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]/2$.