To create is always to learn, to begin over, to begin at zero. – Matthew Fox

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**Problem 1** (Putnam 1974. A1)**.** *Call a set of positive integers "conspiratorial" if no three of them are pairwise relatively prime. (A set of integers is pairwise relatively prime when no pair of them has a common divisor greater than 1.) What is the largest number of elements in any conspiratorial subset of the integers 1 through 16?*

**Problem 2** (Putnam 1967. A1)**.** *Let* $f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx$*, where* $a_1, a_2, \ldots, a_n$ *are real numbers and where* $n$ *is a positive integer. Given that* $|f(x)| \leq |\sin x|$ *for all real* $x$*, prove that* $|a_1 + 2a_2 + \cdots + na_n| \leq 1$*.*

**Problem 3** (Putnam 1968. A3)**.** *Prove that a list can be made of all the subsets of a finite set in such a way that (i) the empty set is first in the list, (ii) each subset occurs exactly once, and (iii) each subset in the list is obtained either by adding one element to the preceding subset or by deleting one element of the preceding subset.*

**Problem 4** (Putnam 1970. B2)**.** *The time-varying temperature of a certain body is given by a polynomial in the time, of degree* $\leq 3$*. Show that the average temperature of the body between 9am and 3pm can always be found by taking the average of the temperatures at two fixed times (10:16am and 1:44pm to the nearest minute), independent of the polynomial.*

**Problem 5** (Putnam 1976. B1)**.** *Evaluate* $\displaystyle\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left( \left\lfloor \frac{2n}{k} \right\rfloor - 2 \left\lfloor \frac{n}{k} \right\rfloor \right)$.

**Problem 6** (Putnam 1973. B5)**.**

*(a) Let* $z$ *be a solution of the quadratic* $az^2 + bz + c = 0$ *and let* $n$ *be a positive integer. Show that* $z$ *can be expressed as a rational function of* $z^n, a, b, c$*.*

*(b) Express* $x$ *as a rational function of* $x^3$ *and* $x + 1/x$*.*

Hints:

1. Try playing around with larger subsets. Are there any specific collections of numbers that aren't allowed to be included together?

2. What does $f(x)/\sin x$ look like? Can you say anything useful about its behavior, for instance as $x \to 0$?

3. Try this out with small sets. You can equivalently pose this as a question about graphs or binary sequences, which might be more helpful.

4. It's easier to see what's going on if you reformat the question to be about polynomials on $[-1, 1]$ by scaling linearly.

5. A limit of $1/n$ times a sum of $n$ terms, each from a function evaluation, is a Riemann sum. What Riemann sum is this? Can you evaluate the integral?

6. For (a) try using induction to show that if $z^2 = rz + s$ then $z^n = r_n z + s_n$ for some polynomials $r_n, s_n$ in $r, s$. For (b) work out the recurrence explicitly.